The College Board
Advanced Placement Examination
CALCULUS AB
SECTION II

This green insert may be used for reference and/or scratchwork as you answer the free-response questions, but be sure to show all your work and your answers in the <u>pink</u> booklet. No credit will be given for work shown on this green insert.

For face-to-face teaching purposes, classroom teachers are permitted to reproduce <u>only</u> the questions in this green insert.

CALCULUS AB

SECTION II

Time — 1 hour and 30 minutes

Number of problems — 6

Percent of total grade — 50

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR
PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

General instructions for this section are printed on the back cover of the test booklet.

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1. Let f be the function given by $f(x) = \dfrac{2x}{\sqrt{x^2 + x + 1}}$.

   (a) Find the domain of f. Justify your answer.

   (b) In the viewing window provided below, sketch the graph of f.
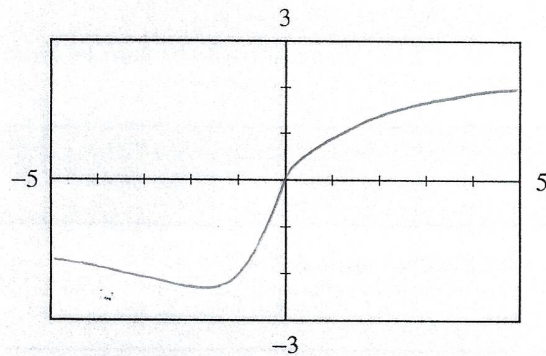
Viewing Window
$[-5, 5] \times [-3, 3]$

   (c) Write an equation for each horizontal asymptote of the graph of f.

   (d) Find the range of f. Use $f'(x)$ to justify your answer.

   Note: $f'(x) = \dfrac{x + 2}{(x^2 + x + 1)^{\frac{3}{2}}}$


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2. A particle moves along the y-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = t \cos t$. At time $t = 0$, the position of the particle is $y = 3$.

(a) For what values of t, $0 \leq t \leq 5$, is the particle moving upward?

(b) Write an expression for the acceleration of the particle in terms of t.

(c) Write an expression for the position $y(t)$ of the particle.

(d) For $t > 0$, find the position of the particle the first time the velocity of the particle is zero.

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3. Consider the curve defined by $-8x^2 + 5xy + y^3 = -149$.

(a) Find $\dfrac{dy}{dx}$.

(b) Write an equation for the line tangent to the curve at the point $(4, -1)$.

(c) There is a number k so that the point $(4.2, k)$ is on the curve. Using the tangent line found in part (b), approximate the value of k.

(d) Write an equation that can be solved to find the actual value of k so that the point $(4.2, k)$ is on the curve.

(e) Solve the equation found in part (d) for the value of k.
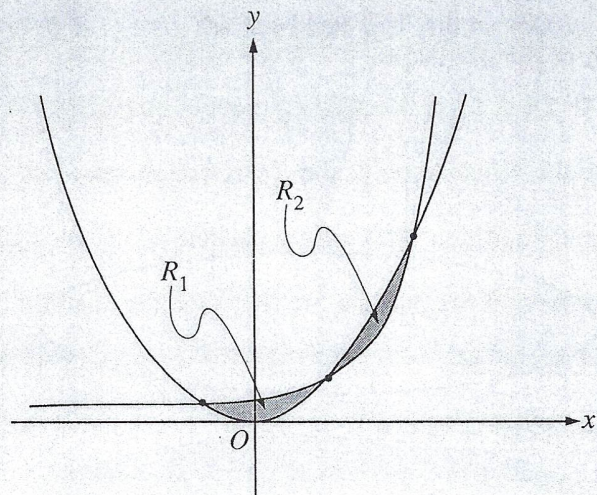
Note: Figure not drawn to scale.

4. The shaded regions R_1 and R_2 shown above are enclosed by the graphs of $f(x) = x^2$ and $g(x) = 2^x$.

(a) Find the x- and y-coordinates of the three points of intersection of the graphs of f and g.

(b) Without using absolute value, set up an expression involving one or more integrals that gives the total area enclosed by the graphs of f and g. Do not evaluate.

(c) Without using absolute value, set up an expression involving one or more integrals that gives the volume of the solid generated by revolving the region R_1 about the line $y = 5$. Do not evaluate.
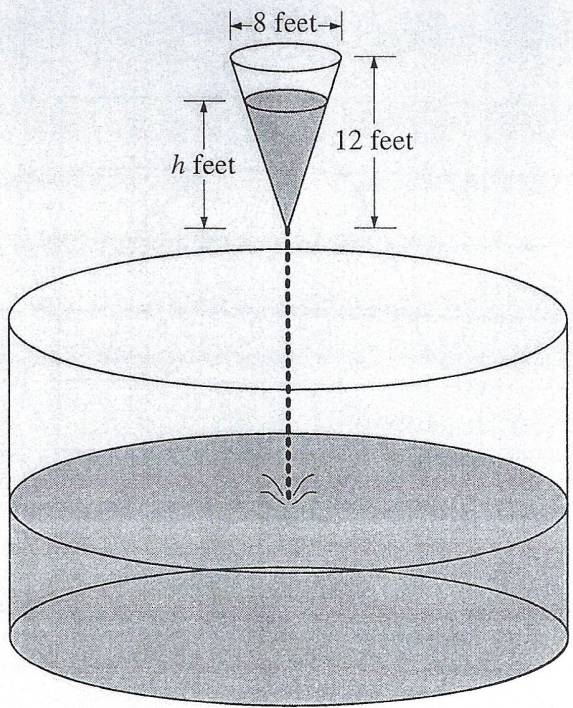
5. As shown in the figure above, water is draining from a conical tank with height 12 feet and diameter 8 feet into a cylindrical tank that has a base with area 400π square feet. The depth h, in feet, of the water in the conical tank is changing at the rate of $(h - 12)$ feet per minute. (The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

(a) Write an expression for the volume of water in the conical tank as a function of h.

(b) At what rate is the volume of water in the conical tank changing when $h = 3$? Indicate units of measure.

(c) Let y be the depth, in feet, of the water in the cylindrical tank. At what rate is y changing when $h = 3$? Indicate units of measure.